1.2 Solving Quadratic Equations Using the Factor Method

Solve the following quadratic equations. (1 - 6)

1. \((x - 1)(x - 2) = 0\)

\[\text{Solution}\]
\[(x - 1)(x - 2) = 0\]
\[x - 1 = 0 \quad \text{or} \quad x - 2 = 0\]
\[x = 1 \quad \text{or} \quad x = 2\]

2. \((x - 7)(x + 8) = 0\)

\[\text{Solution}\]
\[(x - 7)(x + 8) = 0\]
\[x - 7 = 0 \quad \text{or} \quad x + 8 = 0\]
\[x = 7 \quad \text{or} \quad x = -8\]

3. \((4x + 1)(x + 2) = 0\)

\[\text{Solution}\]
\[(4x + 1)(x + 2) = 0\]
\[4x + 1 = 0 \quad \text{or} \quad x + 2 = 0\]
\[x = \frac{-1}{4} \quad \text{or} \quad x = -2\]

4. \(5x(x + 9) = 0\)

\[\text{Solution}\]
\[5x(x + 9) = 0\]
\[x(x + 9) = 0\]
\[x = 0 \quad \text{or} \quad x + 9 = 0\]
\[x = 0 \quad \text{or} \quad x = -9\]
5. \(3(x + 12)(x - 15) = 0\)

**Solution**

\(3(x + 12)(x - 15) = 0\)

\((x + 12)(x - 15) = 0\)

\(x + 12 = 0\) or \(x - 15 = 0\)

\(x = -12\) or \(x = 15\)

8. \(32y - 8y^2 = 0\)

**Solution**

\(32y - 8y^2 = 0\)

\(8y(4 - y) = 0\)

\(y = 0\) or \(4 - y = 0\)

\(y = 0\) or \(y = 4\)

6. \(4(3x - 1)(x + 9) = 0\)

**Solution**

\(4(3x - 1)(x + 9) = 0\)

\((3x - 1)(x + 9) = 0\)

\(3x - 1 = 0\) or \(x + 9 = 0\)

\(x = \frac{1}{3}\) or \(x = -9\)

9. \(x^2 + 11x + 10 = 0\)

**Solution**

\(x^2 + 11x + 10 = 0\)

\([x + (\frac{1}{3})][x + (\frac{1}{1})] = 0\)

\(x + (\frac{1}{3}) = 0\) or \(x + (\frac{1}{1}) = 0\)

\(x = (\frac{-10}{3})\) or \(x = (\frac{-1}{1})\)

7. \(x^2 + 17x = 0\)

**Solution**

\((x)(x + 17) = 0\)

\(x = (0)\) or \(x + 17 = (0)\)

\(x = 0\) or \(x = (\frac{-17}{3})\)

10. \(x^2 + 3x - 10 = 0\)

**Solution**

\(x^2 + 3x - 10 = 0\)

\((x + 5)(x - 2) = 0\)

\(x + 5 = 0\) or \(x - 2 = 0\)

\(x = -5\) or \(x = 2\)
11. \(4y^2 + y - 3 = 0\)

**Solution**

\[
4y^2 + y - 3 = 0
\]

\((4y - 3)(y + 1) = 0\)

\(4y - 3 = 0\) or \(y + 1 = 0\)

\(y = \frac{3}{4}\) or \(y = -1\)

---

12. \(3y^2 = 35y + 12\)

**Solution**

\[3y^2 = 35y + 12\]

\[3y^2 - 35y - 12 = 0\]

\((3y + 1)(y - 12) = 0\)

\(3y + 1 = 0\) or \(y - 12 = 0\)

\(y = -\frac{1}{3}\) or \(y = 12\)

---

Form a quadratic equation in \(x\) with the given roots. (13 - 15)

13. 4, 5

**Solution**

\[
\therefore\ \ x = \left(\ 4 \ \right)\ \text{or}\ \ x = \left(\ 5 \ \right)
\]

\(x - (4) = 0\) or \(x - (5) = 0\)

\[
\therefore\ \ (x - 4)(x - 5) = 0
\]

\(x^2 - 9x + 20 = 0\)

\[
\therefore\ \ \text{The required equation is}\ \ x^2 - 9x + 20 = 0.
\]

14. 9, -1

**Solution**

\[
\therefore\ \ x = 9\ \text{or}\ \ x = -1
\]

\(x - 9 = 0\) or \(x + 1 = 0\)

\[
\therefore\ \ (x - 9)(x + 1) = 0
\]

\(x^2 - 8x - 9 = 0\)

\[
\therefore\ \ \text{The required equation is}\ \ x^2 - 8x - 9 = 0.
\]
15. $10, -\frac{3}{2}$

**Solution**

\[
\begin{align*}
\therefore \quad x &= 10 \quad \text{or} \quad x = -\frac{3}{2} \\
x - 10 &= 0 \quad \text{or} \quad x + \frac{3}{2} = 0 \\
(x - 10)(x + \frac{3}{2}) &= 0 \\
x^2 - \frac{17}{2}x - 15 &= 0 \\
2x^2 - 17x - 30 &= 0 \\
\therefore \quad \text{The required equation is} \\
2x^2 - 17x - 30 &= 0.
\end{align*}
\]

Solve the following quadratic equations.

(16 - 19)

16. $5z^2 - 4(2z + 1) = 0$

**Solution**

\[
\begin{align*}
5z^2 - 4(2z + 1) &= 0 \\
5z^2 - 8z - 4 &= 0 \\
(5z + 2)(z - 2) &= 0 \\
5z + 2 &= 0 \quad \text{or} \quad z - 2 = 0 \\
z &= -\frac{2}{5} \quad \text{or} \quad z = 2
\end{align*}
\]

17. $5(3z + 1) = -4(z^2 + 1)$

**Solution**

\[
\begin{align*}
5(3z + 1) &= -4(z^2 + 1) \\
15z + 5 &= -4z^2 - 4 \\
4z^2 + 15z + 9 &= 0 \\
(4z + 3)(z + 3) &= 0 \\
4z + 3 &= 0 \quad \text{or} \quad z + 3 = 0 \\
z &= -\frac{3}{4} \quad \text{or} \quad z = -3
\end{align*}
\]

18. $(y - 4)^2 + (y - 4) - 30 = 0$

**Solution**

\[
\begin{align*}
(y - 4)^2 + (y - 4) - 30 &= 0 \\
y^2 - 8y + 16 + y - 4 - 30 &= 0 \\
y^2 - 7y - 18 &= 0 \\
(y + 2)(y - 9) &= 0 \\
y + 2 &= 0 \quad \text{or} \quad y - 9 = 0 \\
y &= -2 \quad \text{or} \quad y = 9
\end{align*}
\]

Alternative method:

$(y - 4)^2 + (y - 4) - 30 = 0$

Let $x = y - 4$.

\[
\begin{align*}
x^2 + x - 30 &= 0 \\
(x + 6)(x - 5) &= 0 \\
x + 6 &= 0 \quad \text{or} \quad x - 5 = 0 \\
x &= -6 \quad \text{or} \quad x = 5 \\
y - 4 &= -6 \quad \text{or} \quad y - 4 = 5 \\
y &= -2 \quad \text{or} \quad y = 9
\end{align*}
\]
19. \((4x + 5)^2 = (4x + 5)(x - 2)\)

**Solution**

\[(4x + 5)^2 = (4x + 5)(x - 2)\]

\[(4x + 5)^2 - (4x + 5)(x - 2) = 0\]

\[(4x + 5)[(4x + 5) - (x - 2)] = 0\]

\[(4x + 5)[4x + 5 - x + 2] = 0\]

\[(4x + 5)(3x + 7) = 0\]

\[4x + 5 = 0\quad \text{or}\quad 3x + 7 = 0\]

\[x = - \frac{5}{4}\quad \text{or}\quad x = - \frac{7}{3}\]

20. (a) Solve \(x^2 - 4x - 12 = 0\).

(b) Find a quadratic equation in \(x\) whose roots are half of the roots of \(x^2 - 4x - 12 = 0\).

**Solution**

(a) \(x^2 - 4x - 12 = 0\)

\[(x + 2)(x - 6) = 0\]

\[x + 2 = 0\quad \text{or}\quad x - 6 = 0\]

\[x = -2\quad \text{or}\quad x = 6\]

(b) Half of the roots of the quadratic equation \(x^2 - 4x - 12 = 0\) are \(- \frac{2}{2} = -1\) and \(\frac{6}{2} = 3\).

\[\therefore\quad x = -1\quad \text{or}\quad x = 3\]

\[x + 1 = 0\quad \text{or}\quad x - 3 = 0\]

\[\therefore\quad (x + 1)(x - 3) = 0\]

\[x^2 - 2x - 3 = 0\]

\[\therefore\quad \text{The required equation is } x^2 - 2x - 3 = 0.\]
21. (a) Solve \( x^2 + 5x + 6 = 0 \).

(b) Find a quadratic equation in \( x \) whose roots are 2 and the sum of the roots of \( x^2 + 5x + 6 = 0 \).

**Solution**

(a) \( x^2 + 5x + 6 = 0 \)

\( (x + 2)(x + 3) = 0 \)

\( x + 2 = 0 \) \text{ or } \( x + 3 = 0 \)

\( x = -2 \) \text{ or } \( x = -3 \)

(b) Sum of the roots of \( x^2 + 5x + 6 = 0 \) is \((-2) + (-3) = -5\).

\( \therefore \) \( x = 2 \) \text{ or } \( x = -5 \)

\( x - 2 = 0 \) \text{ or } \( x + 5 = 0 \)

\( \therefore \) \( (x - 2)(x + 5) = 0 \)

\( x^2 + 3x - 10 = 0 \)

\( \therefore \) The required equation is \( x^2 + 3x - 10 = 0 \).

22. If the roots of \( x^2 + bx - 1 = 0 \) have the same numerical value but different signs, find the value of \( b \).

**Solution**

Let \( a \) and \(-a\) be the roots of \( x^2 + bx - 1 = 0 \).

\( x^2 + bx - 1 = (x - a)(x + a) \)

\( = x^2 - a^2 \)

\( \therefore \) \( b = 0 \)
1.3 Solving Quadratic Equations by Forming Perfect Squares

Key Concepts and Formulae

For a quadratic equation in the form of \((x + m)^2 = n\), where \(m\) and \(n\) are real numbers and \(n \geq 0\), the roots are \(x = -m \pm \sqrt{n}\).

Solve the following quadratic equations. (Leave your answers in surd form if necessary.) (1 - 4)

1. \((x + 6)^2 = 16\)

   **Solution**

   
   \((x + 6)^2 = 16\)

   \(x + 6 = \pm 4\)

   \(x = -2\) \hspace{1cm} \text{or} \hspace{1cm} -10 \)

2. \((x - 5)^2 = 9\)

   **Solution**

   
   \((x - 5)^2 = 9\)

   \(x - 5 = \pm 3\)

   \(x = 8\) \hspace{1cm} \text{or} \hspace{1cm} 2 \)

3. \(\left(x - \frac{8}{5}\right)^2 = 81\)

   **Solution**

   
   \(\left(x - \frac{8}{5}\right)^2 = 81\)

   \(x - \frac{8}{5} = \pm 9\)

   \(\therefore x = \frac{53}{5}\) \hspace{1cm} \text{or} \hspace{1cm} -\frac{37}{5}\)

4. \(4\left(x + \frac{3}{10}\right)^2 = 49\)

   **Solution**

   
   \(4\left(x + \frac{3}{10}\right)^2 = 49\)

   \(\left(x + \frac{3}{10}\right)^2 = \frac{49}{4}\)

   \(x + \frac{3}{10} = \pm \frac{7}{2}\)

   \(\therefore x = \frac{16}{5}\) \hspace{1cm} \text{or} \hspace{1cm} -\frac{19}{5}\)
Given that the following expressions are perfect squares, find the value of \( r \). Rewrite the expressions in the form of \((x \pm m)^2\). (5 - 8)

5. \( x^2 + 4x + r \)

Solution
\[
r = \left( \frac{4}{2} \right)^2
= \left( 2 \right)^2
\]
\[
\therefore \ x^2 + 4x + (2)
= (x + 2)^2
\]

6. \( x^2 - 10x + r \)

Solution
\[
r = \left( \frac{-10}{2} \right)^2
= \frac{25}{1}
\]
\[
\therefore \ x^2 - 10x + 25 = (x - 5)^2
\]

7. \( x^2 - 9x + r \)

Solution
\[
r = \left( \frac{-9}{2} \right)^2
= \frac{81}{4}
\]
\[
\therefore \ x^2 - 9x + \frac{81}{4} = \left( x - \frac{9}{2} \right)^2
\]

8. \( x^2 + \frac{7}{2}x + r \)

Solution
\[
r = \left( \frac{7}{4} \right)^2
= \frac{49}{16}
\]
\[
\therefore \ x^2 + \frac{7}{2}x + \frac{49}{16} = \left( x + \frac{7}{4} \right)^2
\]
Solve the following quadratic equations by completing the square. (Leave your answers in surd form if necessary.) (9 - 12)

9. \( x^2 + 4x - 5 = 0 \)

Solution

\[
x^2 + 4x - 5 = 0
\]

\[
x^2 + 4x = 5
\]

\[
x^2 + 4x + \left( \frac{4}{2} \right)^2 = 5 + \left( \frac{4}{2} \right)^2
\]

\[
(x + 2)^2 = (9)
\]

\[
x + (2) = \pm(3)
\]

\[
x = (1) \text{ or } (-5)
\]

10. \( x^2 + 8x - 20 = 0 \)

Solution

\[
x^2 + 8x - 20 = 0
\]

\[
x^2 + 8x = 20
\]

\[
x^2 + 8x + \left( \frac{8}{2} \right)^2 = 20 + \left( \frac{8}{2} \right)^2
\]

\[
(x + 4)^2 = 36
\]

\[
x + 4 = \pm 6
\]

\[
x = 2 \text{ or } -10
\]

11. \( x^2 - 11x + 18 = 0 \)

Solution

\[
x^2 - 11x + 18 = 0
\]

\[
x^2 - 11x = -18
\]

\[
x^2 - 11x + \left( \frac{11}{2} \right)^2 = -18 + \left( \frac{11}{2} \right)^2
\]

\[
\left( x - \frac{11}{2} \right)^2 = \frac{49}{4}
\]

\[
x - \frac{11}{2} = \pm \frac{7}{2}
\]

\[
x = 9 \text{ or } 2
\]

12. \( 6x^2 + 5x + 1 = 0 \)

Solution

\[
6x^2 + 5x + 1 = 0
\]

\[
x^2 + \frac{5}{6}x = -\frac{1}{6}
\]

\[
x^2 + \frac{5}{6}x + \left( \frac{5}{12} \right)^2 = -\frac{1}{6} + \left( \frac{5}{12} \right)^2
\]

\[
\left( x + \frac{5}{12} \right)^2 = \frac{1}{144}
\]

\[
x + \frac{5}{12} = \pm \frac{1}{12}
\]

\[
x = -\frac{1}{2} \text{ or } -\frac{1}{3}
\]
1. Quadratic Equations in One Unknown

**Exercise 1C**

Name: \\
Date: \\
Mark: \\

1.4A Quadratic Formula

**Key Concepts and Formulae**

For a quadratic equation in the general form \( ax^2 + bx + c = 0 \), where \( a, b \) and \( c \) are real numbers and \( a \neq 0 \), the roots are \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \).

Solve the following quadratic equations using the quadratic formula. (1 - 4)

1. \( x^2 + 3x + 2 = 0 \)

**Solution**

\[
x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(2)}}{2(1)}
\]

\[
= \frac{-3 \pm \sqrt{1}}{2}
\]

\[
= \frac{-3 \pm 1}{2}
\]

\[
= \frac{-1}{2} \text{ or } \frac{-5}{2}
\]

2. \( x^2 - x - 56 = 0 \)

**Solution**

\[
x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-56)}}{2(1)}
\]

\[
= \frac{1 \pm \sqrt{225}}{2}
\]

\[
= \frac{1 \pm 15}{2}
\]

\[
= 8 \text{ or } -7
\]
3. \( x^2 - 11x - 242 = 0 \)

**Solution**

\[
x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(1)(-242)}}{2(1)}
\]

\[
x = \frac{11 \pm \sqrt{1089}}{2}
\]

\[
x = \frac{11 \pm 33}{2}
\]

\[
x = 22 \text{ or } -11
\]

4. \( 6x^2 - x - 40 = 0 \)

**Solution**

\[
x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(6)(-40)}}{2(6)}
\]

\[
x = \frac{1 \pm \sqrt{961}}{12}
\]

\[
x = \frac{1 \pm 31}{12}
\]

\[
x = \frac{8}{3} \text{ or } -\frac{5}{2}
\]

Solve the following quadratic equations using the quadratic formula. (Leave your answers in surd form.) (5 - 7)

5. \( 5x^2 + 9x + 2 = 0 \)

**Solution**

\[
x = \frac{-9 \pm \sqrt{9^2 - 4(5)(2)}}{2(5)}
\]

\[
x = \frac{-9 \pm \sqrt{41}}{10}
\]

6. \( 13x^2 - 4x - 1 = 0 \)

**Solution**

\[
x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(13)(-1)}}{2(13)}
\]

\[
x = \frac{4 \pm \sqrt{68}}{26}
\]

\[
x = \frac{4 \pm 2\sqrt{17}}{26}
\]

\[
x = \frac{2 \pm \sqrt{17}}{13}
\]
7. \(11x^2 + 2x - 8 = 0\)

**Solution**

\[ x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-8)}}{2(1)} \]
\[ x = \frac{-2 \pm \sqrt{36}}{2} \]
\[ x = \frac{-2 \pm 6}{2} \]
\[ x = 2 \text{ or } x = -4 \]

9. \((x + 12)x = 23\)

**Solution**

\( (x + 12)x = 23 \)

\[ x^2 + 12x - 23 = 0 \]

\[ x = \frac{-12 \pm \sqrt{12^2 - 4(1)(-23)}}{2(1)} \]
\[ x = \frac{-12 \pm \sqrt{36 + 92}}{2} \]
\[ x = \frac{-12 \pm \sqrt{128}}{2} \]
\[ x = \frac{-12 \pm 8\sqrt{2}}{2} \]
\[ x = -6 \pm 4\sqrt{2} \]
\[ x = 1.68 \text{ (cor. to 2 d.p.) or } x = -13.68 \text{ (cor. to 2 d.p.)} \]

Solve the following quadratic equations using the quadratic formula. (Give your answers correct to 2 decimal places.) (8 - 10)

8. \(2x - 10 = -7x^2\)

**Solution**

\[ 2x - 10 = -7x^2 \]
\[ 7x^2 + 2x - 10 = 0 \]

\[ x = \frac{-2 \pm \sqrt{2^2 - 4(7)(-10)}}{2(7)} \]
\[ x = \frac{-2 \pm \sqrt{4 + 280}}{14} \]
\[ x = \frac{-2 \pm \sqrt{284}}{14} \]
\[ x = 1.06 \text{ (cor. to 2 d.p.) or } x = -1.35 \text{ (cor. to 2 d.p.)} \]

10. \((x + 12)(x - 1) = x + 28\)

**Solution**

\( (x + 12)(x - 1) = x + 28 \)

\[ x^2 + 10x - 40 = 0 \]

\[ x = \frac{-10 \pm \sqrt{10^2 - 4(1)(-40)}}{2(1)} \]
\[ x = \frac{-10 \pm \sqrt{100 + 160}}{2} \]
\[ x = \frac{-10 \pm \sqrt{260}}{2} \]
\[ x = \frac{-10 \pm 16.12}{2} \]
\[ x = 3.06 \text{ (cor. to 2 d.p.) or } x = -13.06 \text{ (cor. to 2 d.p.)} \]
1.4B Nature of Roots

Find the discriminant for each of the following quadratic equations and determine the nature of the roots. (1 - 6)

1. \(x^2 + 2x - 3 = 0\)

**Solution**

\[\Delta = (2)^2 - 4(1)(-3)\]
\[= 4 + 12\]
\[\therefore \Delta = 16 > 0\]
\[\therefore \text{The equation has two distinct real roots.}\]

2. \(x^2 - x + 10 = 0\)

**Solution**

\[\Delta = (-1)^2 - 4(1)(10)\]
\[= -39\]
\[\therefore \Delta < 0\]
\[\therefore \text{The equation has no real roots.}\]

3. \(6x^2 + 8x - 21 = 0\)

**Solution**

\[\Delta = 8^2 - 4(6)(-21)\]
\[= 568\]
\[\therefore \Delta > 0\]
\[\therefore \text{The equation has two distinct real roots.}\]

4. \(4x^2 + 10x + 1 = 0\)

**Solution**

\[\Delta = 10^2 - 4(4)(1)\]
\[= 84\]
\[\therefore \Delta > 0\]
\[\therefore \text{The equation has two distinct real roots.}\]
5. \( 12x^2 - 5x + 3 = 0 \)

**Solution**

\[
\Delta = (-5)^2 - 4(12)(3) \\
= -119 \\
\therefore \Delta < 0 \\
\therefore \text{The equation has no real roots.}
\]

6. \( 4x^2 - 4x + 1 = 0 \)

**Solution**

\[
\Delta = (-4)^2 - 4(4)(1) \\
= 0 \\
\therefore \Delta = 0 \\
\therefore \text{The equation has a double real root.}
\]

For each of the following quadratic equations, find the value or the range of possible values of \( m \) with the given nature of roots. (7 - 12)

7. \( x^2 + 8x + 2m = 0 \) has two distinct real roots.

**Solution**

\[
\therefore \Delta (\text{激} / = / < ) 0 \\
(8)^2 - 4(1)(2m) (\text{激} / = / < ) 0 \\
64 - 8m (\text{激} / = / < ) 0 \\
m > 8 \\
\therefore \text{The range of possible values of } m \text{ is } (m < 8).
\]

8. \( 4x^2 + 12x + 3m = 0 \) has a double real root.

**Solution**

\[
\therefore \text{The equation } 4x^2 + 12x + 3m = 0 \text{ has a double real root.} \\
\therefore \Delta = 0 \\
12^2 - 4(4)(3m) = 0 \\
144 - 48m = 0 \\
m = 3
\]
9. \[ mx^2 - 4x + 16 = 0 \] has no real roots.

**Solution**

\[ \Delta < 0 \]
\[ (-4)^2 - 4(m)(16) < 0 \]
\[ 16 - 64m < 0 \]
\[ m > \frac{1}{4} \]

\[ \because \text{The range of possible values of } m \text{ is } m > \frac{1}{4}. \]

10. \[ 25x^2 + mx + 1 = 0 \] has a double real root.

**Solution**

\[ \Delta = 0 \]
\[ m^2 - 4(25)(1) = 0 \]
\[ m^2 - 100 = 0 \]
\[ m^2 = 100 \]
\[ m = 10 \text{ or } m = -10 \]

11. \[ 36x^2 - 14x + m = 0 \] has real roots.

**Solution**

\[ \Delta \geq 0 \]
\[ (-14)^2 - 4(36)(m) \geq 0 \]
\[ 196 - 144m \geq 0 \]
\[ m \leq \frac{49}{36} \]

\[ \because \text{The range of possible values of } m \text{ is } m \leq \frac{49}{36}. \]
12. \(2mx^2 + 7x - 4 = 0\) has no real roots.

**Solution**

\[ \therefore \text{The equation } 2mx^2 + 7x - 4 = 0 \text{ has no real roots.} \]

\[ \therefore \Delta < 0 \]

\[ 7^2 - 4(2m)(-4) < 0 \]

\[ 49 + 32m < 0 \]

\[ m < -\frac{49}{32} \]

\[ \therefore \text{The range of possible values of } m \text{ is } m < -\frac{49}{32}. \]

13. If the quadratic equation \(16x^2 + 7px + 49 = 0\) has one double real root, find

(a) the values of \(p\),

(b) the root of the equation for each value of \(p\) in (a).

**Solution**

(a) \[ \therefore \text{The equation } 16x^2 + 7px + 49 = 0 \text{ has one double real root.} \]

\[ \therefore \Delta = 0 \]

\[ (7p)^2 - 4(16)(49) = 0 \]

\[ 49p^2 = 3136 \]

\[ p^2 = 64 \]

\[ \therefore p = 8 \text{ or } p = -8 \]

(b) For \(p = 8\), the equation becomes

\[ 16x^2 + 56x + 49 = 0 \]

\[ (4x + 7)^2 = 0 \]

\[ \therefore x = -\frac{7}{4} \]

For \(p = -8\), the equation becomes

\[ 16x^2 - 56x + 49 = 0 \]

\[ (4x - 7)^2 = 0 \]

\[ \therefore x = \frac{7}{4} \]
14. It is given that $10x^2 - 2x + p = 0$ has real roots.

(a) Find the range of possible values of $p$.

(b) From the result obtained in (a), determine whether the following equations have real roots.

(i) $10x^2 - 2x + 1 = 0$

(ii) $5x^2 - x - 1 = 0$

Solution

(a) $\therefore$ The equation $10x^2 - 2x + p = 0$ has real roots.

$\therefore \Delta \geq 0$

$(-2)^2 - 4(10)(p) \geq 0$

$4 - 40p \geq 0$

$p \leq \frac{1}{10}$

(b) (i) $p = 1$

$> \frac{1}{10}$

$\therefore$ The equation $10x^2 - 2x + 1 = 0$ has no real roots.

(ii) Multiplying both sides of the equation $5x^2 - x - 1 = 0$ by 2, we have

$10x^2 - 2x - 2 = 0$

$p = -2$

$\leq \frac{1}{10}$

$\therefore$ The equation $5x^2 - x - 1 = 0$ has real roots.
1.5 Solving Quadratic Equations Using the Graphical Method

Key Concepts and Formulae

1. Plot the graph of $y = ax^2 + bx + c$. The $x$-intercepts of the quadratic graph are the roots of the equation $ax^2 + bx + c = 0$. However, the values of $x$-intercepts read from the graph are approximations only.

2. $\Delta = b^2 - 4ac$

<table>
<thead>
<tr>
<th>$\Delta &gt; 0$</th>
<th>$\Delta = 0$</th>
<th>$\Delta &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two $x$-intercepts</td>
<td>Only one $x$-intercept</td>
<td>No $x$-intercept</td>
</tr>
</tbody>
</table>
1. The figure shows the graph of \( y = x^2 - 3x - 4 \).

(a) How many real roots does the equation \( x^2 - 3x - 4 = 0 \) have?

(b) Solve the equation \( x^2 - 3x - 4 = 0 \) graphically.

**Solution**

(a) The graph of \( y = x^2 - 3x - 4 \) has \( \text{two} \) \( x \)-intercepts. Therefore, the quadratic equation \( x^2 - 3x - 4 = 0 \) has \( \text{two} \) real roots.

(b) The \( x \)-intercepts of \( y = x^2 - 3x - 4 \) are \( -1 \) and \( 4 \). Therefore, the roots of the equation are \( -1 \) and \( 4 \).

2. The figure shows the graph of \( y = x^2 - 7x + 10 \).

(a) How many real roots does the equation \( x^2 - 7x + 10 = 0 \) have?

(b) Solve the equation \( x^2 - 7x + 10 = 0 \) graphically.

**Solution**

(a) The graph of \( y = x^2 - 7x + 10 \) has two \( x \)-intercepts. Therefore, the quadratic equation \( x^2 - 7x + 10 = 0 \) has two real roots.

(b) The \( x \)-intercepts of \( y = x^2 - 7x + 10 \) are 2 and 5. Therefore, the roots of the equation \( x^2 - 7x + 10 = 0 \) are 2 and 5.
3. The figure shows the graph of $y = x^2 + 8x + 16$.

(a) How many real roots does the equation $x^2 + 8x + 16 = 0$ have?

(b) Solve the equation $x^2 + 8x + 16 = 0$ graphically.

Solution

(a) The graph of $y = x^2 + 8x + 16$ has one $x$-intercept.

Therefore, the quadratic equation $x^2 + 8x + 16 = 0$ has a double real root.

(b) The $x$-intercept of $y = x^2 + 8x + 16$ is $-4$.

Therefore, the root of the equation $x^2 + 8x + 16 = 0$ is $-4$.

4. (a) Given that $y = x^2 + 5x + 6$, complete the table for the corresponding values of $y$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-5$</th>
<th>$-4$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Plot the graph of $y = x^2 + 5x + 6$ from $x = -5$ to $x = 0$.

(c) Hence, solve the equation $x^2 + 5x + 6 = 0$ graphically.

Solution

(a)

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-5$</th>
<th>$-4$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>
Quadratic Equations in One Unknown

(b) \[ y = x^2 + 5x + 6 \]

(c) The \( x \)-intercepts of \( y = x^2 + 5x + 6 \) are \((-2)\) and \((-3)\).

Therefore, the roots of the equation \( x^2 + 5x + 6 = 0 \) are \((-2)\) and \((-3)\).

5. (a) Plot the graph of \( y = x^2 - x - 12 \) from \( x = -4 \) to \( x = 5 \).
[Suggestion: Scale for \( x \)-axis is 10 divisions (1 cm) = 1 unit;
scale for \( y \)-axis is 10 divisions (1 cm) = 2 units]

(b) Hence, solve the equation \( x^2 - x - 12 = 0 \) graphically.

Solution

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>8</td>
<td>0</td>
<td>-6</td>
<td>-10</td>
<td>-12</td>
<td>-12</td>
<td>-10</td>
<td>-6</td>
<td>0</td>
<td>8</td>
</tr>
</tbody>
</table>
(b) The $x$-intercepts of $y = x^2 - x - 12$ are $-3$ and $4$.
Therefore, the roots of the equation $x^2 - x - 12 = 0$ are $-3$ and $4$. 
6. (a) Plot the graph of \( y = -x^2 + 2x + 6 \) from \( x = -3 \) to \( x = 5 \).
[Suggestion: Scale for \( x \)-axis is 10 divisions (1 cm) = 1 unit; scale for \( y \)-axis is 10 divisions (1 cm) = 2 units]

(b) Hence, solve the equation \( x^2 - 6 = 2x \) graphically.

**Solution**

(a) \[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
x & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
y & -9 & -2 & 3 & 6 & 7 & 6 & 3 & -2 & -9 \\
\hline
\end{array}
\]

(b) \( x^2 - 6 = 2x \)

\[-x^2 + 2x + 6 = 0\]

The \( x \)-intercepts of \( y = -x^2 + 2x + 6 \) are \(-1.6\) and \(3.6\).

Therefore, the roots of the equation \( x^2 - 6 = 2x \) are \(-1.6\) and \(3.6\).
7. The graph of \( y = 6x^2 + 5x + q \) does not intersect the \( x \)-axis. Find the range of possible values of \( q \).

**Solution**

The graph of \( y = 6x^2 + 5x + q \) does not intersect the \( x \)-axis.

Therefore, \( \Delta < 0 \)

i.e. \( 5^2 - 4(6)(q) < 0 \)

\[
25 - 24q < 0
\]

\[
q > \frac{25}{24}
\]

\( \therefore \) The range of possible values of \( q \) is \( q > \frac{25}{24} \).

8. The graph of \( y = x^2 - 12x + p \) intersects the \( x \)-axis.

(a) Find the range of possible values of \( p \).

(b) If \( p \) takes the maximum integral value of \( p \) in (a),

   (i) plot the graph of \( y = x^2 - 12x + p \) for \( x = 3 \) to \( x = 9 \).

   [Suggestion: Scale for \( x \)-axis is 10 divisions (1 cm) = 1 unit; scale for \( y \)-axis is 10 divisions (1 cm) = 1 unit]

   (ii) Hence, solve the equation \( x^2 - 12x + p = 0 \) graphically.

**Solution**

(a) The graph of \( y = x^2 - 12x + p \) intersects the \( x \)-axis.

Therefore, \( \Delta ( \geq ) 0 \)

\[
(-12)^2 - 4(1)(p) \geq 0
\]

\[
144 - 4p \geq 0
\]

\[
p \leq 36
\]

\( \therefore \) The range of possible values of \( p \) is \( p \leq 36 \).
(b) (i) The maximum integral value of \( p = ( \quad 36 \quad ) \)

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

(ii) From the graph in (b) (i), \( x^2 - 12x + 36 = 0 \) has a double root of 6.
1. The square of the difference between a number and 2 is equal to the number itself. Find this number.

Solution

Let \( x \) be the number.

\[
(x - 2)^2 = x
\]

\[
x^2 - 4x + 4 = x
\]

\[
x^2 - 5x + 4 = 0
\]

\[
(x - 1)(x - 4) = 0
\]

\[
x - 1 = 0 \text{ or } x - 4 = 0
\]

\[
x = 1 \text{ or } x = 4
\]

\[
\therefore \text{ The number is ( 1 ) or ( 4 ).}
\]
2. Consider two positive odd numbers. Twice the product of the two numbers is larger than the sum of the two numbers by 2. Find the smaller number.

**Solution**

Let \( x \) be the smaller number.

\[
2x(x + 2) = x + (x + 2) + 2
\]

\[
2x^2 + 4x = 2x + 4
\]

\[
2x^2 + 2x - 4 = 0
\]

\[
x^2 + x - 2 = 0
\]

\[
(x - 1)(x + 2) = 0
\]

\( x = 1 \) or \( x = -2 \) (rejected)

\[
\therefore \text{ The smaller number is } 1.
\]

3. The length of \( AB \) and \( AC \) of a right-angled triangle are \((x + 1)\) cm and \((3x + 2)\) cm respectively. If the area of the triangle is \(26\) \(cm^2\), find \( AC \).

**Solution**

\[
\therefore \text{ The area of the triangle is } 26\text{ cm}^2.
\]

\[
\therefore \frac{(x + 1)(3x + 2)}{2} = 26
\]

\[
3x^2 + 5x + 2 = 52
\]

\[
3x^2 + 5x - 50 = 0
\]

\[
(3x - 10)(x + 5) = 0
\]

\( x = \frac{10}{3} \) or \( x = -5 \) (rejected)

\[
\therefore AC = \left[3\left(\frac{10}{3}\right) + 2\right] \text{cm}
\]

\[
= 12 \text{ cm}
\]
4. The length of a rectangular garden is 4 m longer than its width. If the numerical value of the product of the length and the width is greater than half the numerical value of the perimeter of the garden by 44. Find the length and the width of the garden.

**Solution**

Let \(x\) m be the width of the garden, then \((x + 4)\) m is the length of the garden.

\[
x(x + 4) = \frac{2(x + (x + 4))}{2} + 44
\]

\[
x^2 + 4x = 2x + 48
\]

\[
x^2 + 2x - 48 = 0
\]

\[
(x - 6)(x + 8) = 0
\]

\(x = 6\) or \(x = -8\) (rejected)

\(\therefore\) The width and the length of the garden are 6 m and \((6 + 4)\) m = 10 m.

5. A copper wire is cut into three portions. The three portions are bent to form squares \(A\), \(B\) and \(C\). The lengths of sides of squares \(B\) and \(C\) are longer than that of square \(A\) by 6 cm and 12 cm respectively. If twice the area of square \(A\) is smaller than the area of square \(B\) by 8 cm², find the length of the copper wire.

**Solution**

Let \(x\) cm be the length of a side of square \(A\), then \((x + 6)\) cm and \((x + 12)\) cm are the lengths of sides of squares \(B\) and \(C\) respectively.

\[
2x^2 = (x + 6)^2 - 8
\]

\[
2x^2 = x^2 + 12x + 28
\]

\[
x^2 - 12x - 28 = 0
\]

\[(x - 14)(x + 2) = 0
\]

\(x = 14\) or \(x = -2\) (rejected)

\(\therefore\) The length of the copper wire

\[
= 4[x + (x + 6) + (x + 12)] \text{ cm}
\]

\[
= 4[14 + (14 + 6) + (14 + 12)] \text{ cm}
\]

\[
= 240 \text{ cm}
\]
6. Two cars $X$ and $Y$ are 500 km and 300 km away from city $A$. They start travelling to city $A$ at the same time with constant speeds below 100 km/h. If car $X$ reaches city $A$ half an hour later than car $Y$ and the speed of car $Y$ is 15 km/h less than car $X$, find the speeds of the two cars.

**Solution**

Let $x$ km/h be the speed of car $Y$, then $(x + 15)$ km/h is the speed of car $X$.

\[
\begin{align*}
\frac{500}{x + 15} &= \frac{300}{x} + \frac{1}{2} \\
\frac{500}{x + 15} &= \frac{600 + x}{2x}
\end{align*}
\]

$1000x = x^2 + 615x + 9000$

$x^2 - 385x + 9000 = 0$

$(x - 25)(x - 360) = 0$

$x = 25$ or $x = 360$ (rejected)

\[
\therefore \text{The speed of car } X \text{ is } (25 + 15) \text{ km/h} = 40 \text{ km/h} \text{ and the speed of car } Y \text{ is } 25 \text{ km/h}.
\]

7. A shopkeeper brought a certain number of packets of sweets for $600. He sold them at $12/packet and the profit is less than the cost of 40 packets of sweets by $20. Find the cost of each packet of sweets.

**Solution**

Let $x$ be the cost of each packet of sweets.

\[
\begin{align*}
\frac{600}{x} (12) - 600 &= 40x - 20 \\
\frac{7200 - 600x}{x} &= 40x - 20 \\
7200 - 600x &= 40x^2 - 20x
\end{align*}
\]

$40x^2 + 580x - 7200 = 0$

$2x^2 + 29x - 360 = 0$

$(x - 8)(2x + 45) = 0$

$x = 8$ or $x = -\frac{45}{2}$ (rejected)

\[
\therefore \text{The cost of each packet of sweets is } $8.
\]
8. A farmer plans to sell a certain number of eggs for $180. However, 20 eggs are broken. The farmer wants to earn the same amount of money when he sells all the eggs, so he increases the price of each egg by $0.1. Find the original number of eggs.

Solution

Let \( x \) be the original number of eggs.

\[
(x - 20)\left(\frac{180}{x} + 0.1\right) = 180
\]

\[
(x - 20)(180 + 0.1x) = 180x
\]

\[
0.1x^2 - 2x - 3600 = 0
\]

\[
x^2 - 20x - 36000 = 0
\]

\[
(x - 200)(x + 180) = 0
\]

\[x = 200 \text{ or } x = -180 \text{ (rejected)}\]

\[\therefore \text{ The original number of eggs is 200.}\]

9. In the figure, \( ABCD \) is a rectangle of width 6 cm and length 10 cm. \( X \) and \( Y \) are two points on \( AD \) and \( CD \) respectively such that \( AX = (x + 2) \) cm and \( DY = x \) cm. If the area of the shaded region is 22 cm\(^2\), find \( x \).

Solution

\[\therefore \text{ The area of the shaded region is } 22 \text{ cm}^2.\]

\[\therefore 6(10) - \frac{6(x + 2)}{2} - \frac{x[10 - (x + 2)]}{2} - \frac{10(6 - x)}{2} = 22\]

\[x^2 - 4x + 48 = 44\]

\[x^2 - 4x + 4 = 0\]

\[(x - 2)^2 = 0\]

\[x = 2\]
1. What is the value of $a$ if the quadratic equation $7x = 8 - 3x^2$ is written in the general form $ax^2 + bx + c = 0$?
   A. $-3$  
   B. $3$  
   C. $7$  
   D. $8$  
   **B**

2. What is the discriminant of the quadratic equation $x^2 + 12x + 10 = 0$?
   A. $104$  
   B. $184$  
   C. $134$  
   D. $12$  
   **A**

3. How many roots does the quadratic equation $x^2 + 6x - 36 = 0$ have?
   A. $1$  
   B. $0$  
   C. $2$  
   D. $3$  
   **C**

4. Which of the following is the general form of the quadratic equation $2(x^2 + x) = 15x + 3$?
   A. $2(x^2 + x) = 3(5x + 1)$  
   B. $2x^2 - 13x - 3 = 0$  
   C. $2x^2 - 13x + 3 = 0$  
   D. $2x^2 + 3 + 13x = 0$  
   **B**

5. Which of the following quadratic equations have roots $-5$ and $3$?
   A. $x^2 + 8x - 15 = 0$  
   B. $x^2 - 2x - 15 = 0$  
   C. $x^2 - 2x + 15 = 0$  
   D. $x^2 + 2x - 15 = 0$  
   **D**

6. Solve the quadratic equation $x^2 - x - 42 = 0$.
   A. $x = 6$ or $7$  
   B. $x = -6$ or $-7$  
   C. $x = -6$ or $7$  
   D. $x = 6$ or $-7$  
   **C**

7. Solve the quadratic equation $x^2 + 9x + 8 = 0$.
   A. $x = 1$ or $8$  
   B. $x = -1$ or $-8$  
   C. $x = -1$ or $8$  
   D. $x = 1$ or $-8$  
   **B**

8. Which of the following expressions is a perfect square?
   A. $9x^2 + 1 + 9x$  
   B. $25 + 5x + x^2$  
   C. $36x^2 - 12x + 1$  
   D. $4 + 4x^2 - x$  
   **C**

9. If $(x + 2)^2 = 9(x - 5)^2$, then $x =$
   A. $-2$ or $5$.  
   B. $\frac{13}{4}$ or $\frac{17}{2}$.  
   C. $\frac{43}{10}$ or $\frac{47}{8}$.  
   D. $13$  
   **B**
10. If \(6(x - 3) = (x - 3)(x + 5)\), then \(x = \)
A. 3 or 1. B. 3 or -5. C. -3 or 5. D. 6 or 1. A

11. If \((x - 20)\left(\frac{100}{x} + 1\right) = 25\), then \(x = \)

12. Which of the following quadratic equations has a double root \(3\sqrt{2}\) ?
A. \(x^2 - 6\sqrt{2}x + 18 = 0\) B. \(x^2 + 6\sqrt{2}x + 18 = 0\)
C. \(x^2 + 18 = 0\) D. \(x^2 - 18 = 0\) A

13. Which of the following quadratic equations have no real roots?
I. \(2x^2 - 3x + 2 = 0\) 
II. \(5x^2 + 10x + 6 = 0\) 
III. \(x^2 + x + 2 = 0\)
A. I and II only 
B. I and III only 
C. II and III only 
D. I, II and III D

14. If the quadratic equation \(px^2 - 4x + 1 = 0\) has a double real root, then \(p = \)

15. If the quadratic equation \(6x^2 + 14x + (k + 1) = 0\) has two distinct real roots, find the range of possible values of \(k\).
A. \(k > \frac{43}{6}\) B. \(k \geq \frac{43}{6}\)
C. \(k = \frac{43}{6}\) D. \(k < \frac{43}{6}\) D

16. If 0 is one of the roots of the quadratic equation \(ax^2 + 3a^2x - 2a^2 + a + 1 = 0\) in \(x\), then \(a = \)
A. 1 or 2. B. -2 or -1. C. \(-\frac{1}{2}\) or 2. D. 1 or \(-\frac{1}{2}\). D

17. If 4 is one of the roots of the quadratic equation \(2x^2 - 13x + b = 0\), find the other root of the equation.
A. \(\frac{3}{2}\) B. \(\frac{1}{2}\)
C. \(\frac{5}{2}\) D. \(\frac{11}{2}\) C

18. If the quadratic equation \(4x^2 - gx + 15 = 0\) has two distinct real roots, which of the following are the possible values of \(g\)?
I. -16 
II. 14 
III. 18 
A. I and II only 
B. I and III only 
C. II and III only 
D. I, II and III B
19. Which of the following correctly represents the graph of $y = 4x^2 - 5x + 1$?

A. 

B. 

C. 

D. 

20. Which of the following correctly represents the graph of $y = -x^2 + 2x - 1$?

A. 

B. 

C. 

D. 

21. Which of the following equations may represent the graph as shown below?

A. $y = x^2 - x + 2$
B. $y = x^2 - x - 2$
C. $y = -x^2 + x - 2$
D. $y = -x^2 - x - 2$
22. The graph of \( y = 4x^2 - 7x + s \) cuts the \( x \)-axis at points \( P \) and \( Q \) as shown. If one of the roots of \( 4x^2 - 7x + s = 0 \) is 2, find the \( x \)-coordinate of point \( P \).

\[
\begin{align*}
  y &= 4x^2 - 7x + s \\
  x &= 2
\end{align*}
\]

A. \( \frac{-1}{4} \)  
B. \( -2 \)  
C. \( -4 \)  
D. \( \frac{-3}{4} \)  

23. Find the value of \( x \) in the figure.

\[
(2x - 2) \text{ cm} \\
(2x - 1) \text{ cm} \\
(4x - 3) \text{ cm} \\
8 \text{ cm}
\]

A. 3  
B. 4  
C. 5  
D. 6

24. The product of two consecutive positive numbers is greater than their sum by 19, find the two numbers.

A. 2 and 3  
B. 3 and 4  
C. 4 and 5  
D. 5 and 6

25. The area of the following trapezium is 37.5 cm\(^2\), find \( x \).

\[
\begin{align*}
  \text{Area} &= \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height} \\
  37.5 &= \frac{1}{2} \times (\text{top} + \text{bottom}) \times \text{height} \\
  37.5 &= \frac{1}{2} \times ((2x - 1) + (x + 3)) \times 8 \\
  37.5 &= \frac{1}{2} \times (3x + 2) \times 8 \\
  37.5 &= 6x + 4 \\
  x &= \frac{33.5}{6} \\
  x &= 5.58
\end{align*}
\]

A. 2  
B. 3  
C. 4  
D. 5

26. Country \( A \) and Country \( B \) are 8400 km apart. An aeroplane flies from Country \( A \) to Country \( B \) with a constant speed. If the speed of the aeroplane is decreased by 100 km/h, the flight takes 2 hours longer. Find the original speed of the aeroplane.

A. 400 km/h  
B. 500 km/h  
C. 600 km/h  
D. 700 km/h