P.249 (16 to 17)

16. (a) (i) Let R be the radius of the earth, r be the distance between the earth's surface and the space shuttle, g be the gravitational field strength at the earth's surface.

∵ The space shuttle is performing circular motion

∴ gravitational force = centripetal force

$$\frac{GMm}{(R+r)^2} = \frac{mv^2}{(R+r)}$$

$$\frac{GM}{R^2}\times\frac{R^2}{(R+r)} = v^2$$

$$g\times\frac{R^2}{(R+r)} = v^2$$

$$v = R\sqrt{\frac{g}{R+r}}$$

$$= 6.4\times10^6\sqrt{\frac{10}{6.4\times10^6+2.4\times10^5}}$$

$$= 7854.09\text{ m s}^{-1}$$

Remarks: The radius of the Earth should be taken into account in the calculation.

(ii)

Gravitational force acting on an astronaut

$$= -\frac{GMm}{(R+r)^2}$$

$$= -\frac{GM}{R^2}\times\frac{R^2}{(R+r)^2}\times m$$

$$= -g\times\frac{R^2}{(R+r)^2}\times m$$

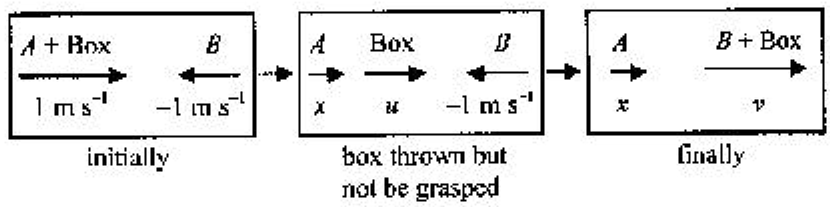
$$= -10\times\frac{(6.4\times10^6)^2}{(6.4\times10^6-2.4\times10^5)^2}\times 60$$

$= -557.41$ N (negative sign means that the force is attractive)

(b) (i) Since the weight of the astronauts is completely used as the centripetal force towards the earth's center, reaction force acting on the astronauts by the floor of the shuttle must be zero, and therefore, they appear to be 'weightless'.

(ii) When A throws the toolbox to B and B grasps it, according to Newton's 3rd Law, A experiences a force in the direction opposite to the motion of the toolbox while B experiences a force in the same direction as the motion of the toolbox. Hence, A and B accelerate in different (opposite) directions. As a result, a head-on collision can be avoided.

(iii)



Consider the above diagram,

By conservation of linear momentum,

$$(60+30)(1) = 60x + 30u \quad \Rightarrow \quad 3 = 2x + u$$

$$30u + 60(-1) = (60+30)v \quad \Rightarrow \quad 3v = u - 2$$

$$\therefore \quad x = \frac{3-u}{2} \quad \text{and} \quad v = \frac{u-2}{3}$$

To avoid collision, we need $x \leq v$,

$$\frac{3-u}{2} \leq \frac{u-2}{3}$$

$$u \geq 2.6 \text{ m s}^{-1}$$

$\therefore$ Minimum speed = 2.6 m s^{-1}

(iv) Workdone on the toolbox by A

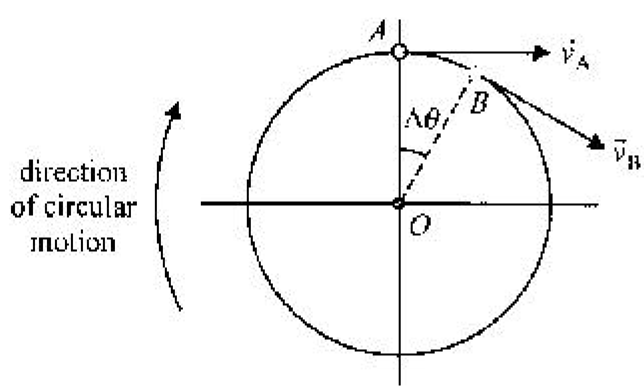
= change in kinetic energy (K.E.) of the toolbox

= K.E. just after A throws the toolbox − K.E. before A throws it

$$= \frac{1}{2}(30)(2.6)^2 - \frac{1}{2}(30)(1)^2$$

= 86.4 J

17. (a) (i) The following diagram shows a particle performing uniform circular motion with speed v on a horizontal surface.

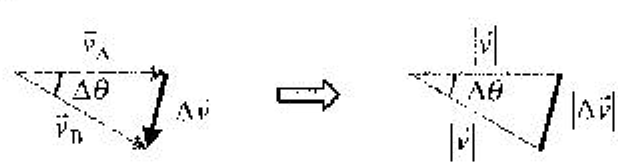


where $\vec{v}_A$ and $\vec{v}_B$ represent the velocity vector of the particle when it is at position A and B respectively. Because the velocity vector is along the circle, $\vec{v}_A \perp OA$ and $\vec{v}_B \perp OB$.

The change in velocity $\Delta\vec{v}$ in a small time interval Δt is given by

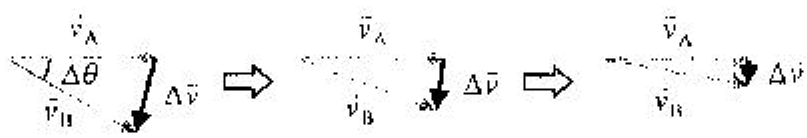
$$\vec{v}_B - \vec{v}_A = \vec{v}_B + (\ \vec{v}_A)$$

The following vector diagram illustrates the direction of change of velocity, $\Delta\vec{v}$.



As the time Δt tends to zero, the angle $\Delta\theta$ also tends to zero. Hence, the magnitude of the change of velocity $\Delta\vec{v} = |\Delta\vec{v}| \approx |\vec{v}|\Delta\theta$, where $|\vec{v}|$ is the magnitude of the uniform speed $\vec{v}$.

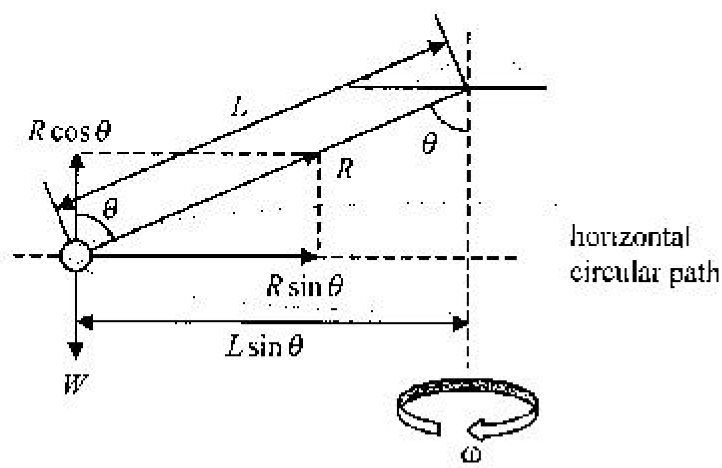
The following diagram shows that, as the time Δt tends to zero, $\vec{v}_A \perp \Delta\vec{v}$. Since $\vec{v}_A \perp OA$, the direction of $\Delta\vec{v}$ points towards the center O.



By $\vec{a} = \lim_{\Delta t\to 0}\frac{\Delta\vec{v}}{\Delta t}$, the direction of the instantaneous acceleration at A is the same as that of the change of velocity, i.e., points to the center O of the circular path.

Also, $|\vec{a}| = \lim_{\Delta t\to 0}\frac{|\Delta\vec{v}|}{\Delta t} = \lim_{\Delta t\to 0}\frac{|\vec{v}|\Delta\theta}{\Delta t} = |\vec{v}|\lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t} = |\vec{v}|\,\omega$

(ii) Let θ be the angle which the string makes with the vertical, and T be the period of the uniform circular motion:



where R is the tension in the string and W is the weight of the pendulum bob.

Consider the forces acting on the pendulum bob along the vertical direction:

$$R\cos\theta = W$$
$$= mg$$

Hence,
$$R = \frac{mg}{\cos\theta} \quad \text{——— (1)}$$

As the pendulum bob is performing horizontal uniform circular motion:

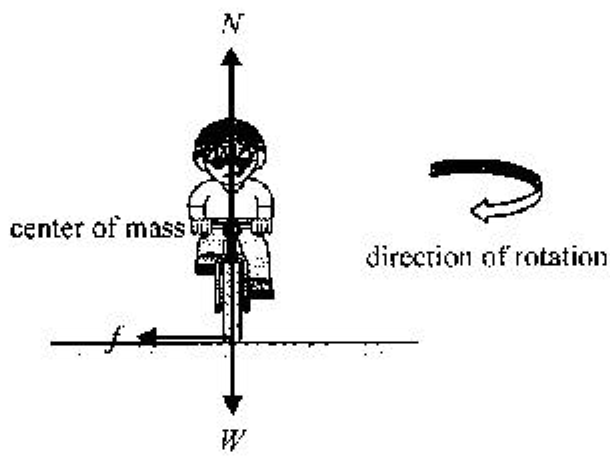
$$R\sin\theta = m\,\omega^2(L\sin\theta)$$
$$R = m\,\omega^2 L$$
$$\frac{mg}{\cos\theta} = m\,\omega^2 L \qquad (\text{from (1)})$$

$$\omega^2 = \frac{g}{L\cos\theta}$$

Hence, $$\omega = \sqrt{\frac{g}{L\cos\theta}}$$

Period of the uniform circular motion = $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$

(iii) The following diagram shows the force diagram of a boy sitting upright on a bicycle and making a circular turn (the center of the circular path is on the left):

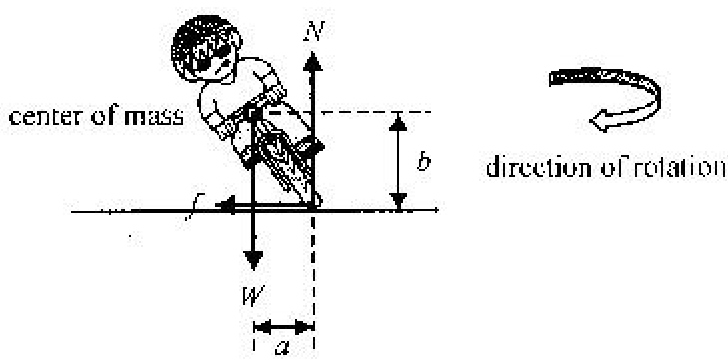


where N = normal reaction by the floor
W = weight of the system (the boy and the bicycle)
f = friction by the floor (for centripetal acceleration)

The line of force of the weight acts through the center of mass of the system, and both the friction and normal reaction acts on the point of the contact between the bicycle and the floor.

The frictional force f causes the boy to turn over (in a clockwise sense), and hence he cannot maintain upright when riding the bicycle round a horizontal circular track.

The following diagram shows the force diagram of the boy leaning inwards for a circular turn:



where N, W and f stands for the same corresponding forces mentioned above. a and b are the perpendicular distances between the line of forces of N and f from the center of mass of the system (the boy and the bicycle).

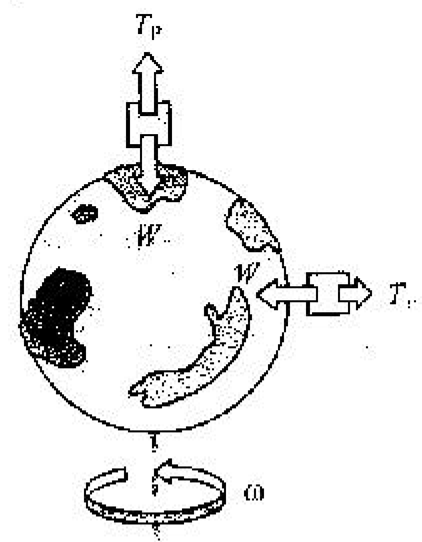
The boy has to lean inward so as to satisfy the following relationship of distances a and b, and hence the system would be in equilibrium.

Taking moments about the center of mass of the system, we have:

$$\text{Total clockwise moment} = \text{Total anti-clockwise moment}$$
$$N \times a = f \times b$$

Therefore, the boy has to lean inwards when riding round a horizontal circular track so that the moment of the centripetal force is balanced by the moment of the normal reaction.

(b) (i) The apparent weight of the object at the pole and Equator are equal to the reading of the spring balance at the corresponding position. Let T_P and T_E be the reading of the spring balance when the object is at the pole and Equator respectively.



When the object is at the pole, net force acting on it is zero since it is stationary with respect to the self-rotation of the Earth. Hence,

$$T_P = W$$

When the object is at the Equator, it is performing uniform circular motion due to the self-rotation of the Earth. Hence,

$$W - T_E = m\omega^2 R_E$$
$$T_E = W - m\omega^2 R_E$$
$$< T_P$$

Therefore, the object's apparent weight when it is located at the Equator is less than that at the pole by $m\omega^2 R_E$.

Remarks: If g' is the acceleration due to gravity at the Equator,

$$g' = g - \omega^2 R_E$$