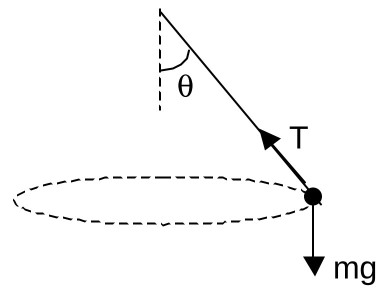
P.196 (6 to 13)

6 (a) From the diagram, T sin è provides the required centripetal force. If the string breaks, no force is available to provide the centripetal acceleration. Therefore, the mass moves in a straight line (i.e. moves at a tangent to the circle.)



(b) The angle θ the pilot should bank is

$$\tan\theta = \frac{v^2}{rg}$$

$$\tan\theta = \frac{\left(360 \times \frac{1\,000}{3\,600}\right)^2}{5\,000 \times 10}$$

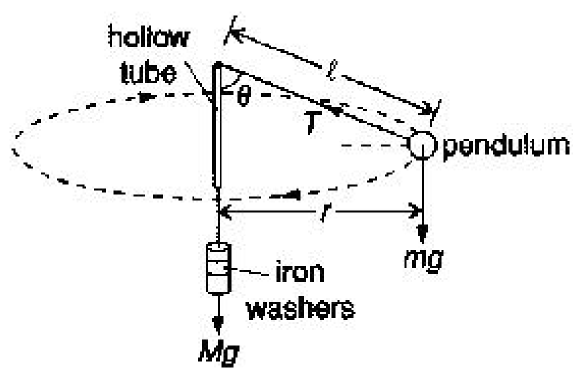
$$\theta = 11.3°$$

7. (a) (i) $a = \frac{v^2}{r}$

(ii) F = m a = m r ù^2

$$= mr\left(\frac{2p}{T}\right)^2 = \frac{4ð^2 mr}{T^2}$$

(b) The experiment is set up as shown. As the pendulum rotates, the tension of the string T is equal to the mass of the iron washers Mg. This provides the required centripetal force F.



F = Mg sinè

From (a) (ii) above, The centripetal force,

$$F = \frac{4ð^2 mr}{T^2}$$

$$\therefore \quad \text{Mg sin è} = \frac{4ð^2 mr}{T^2} = \frac{4ð^2 ml \sin q}{T^2}$$

$$\therefore \quad Mg = \frac{4ð^2 ml}{T^2}$$

By measuring the time taken for 20 revolutions t, we can find the period $T = \frac{t}{20}$. We can also measure M, m and *l*. Substituting the values into the above equation, we can verify experimentally the relationship between F and *T*.

(c) **(i)** The magnitude of the centripetal force F is

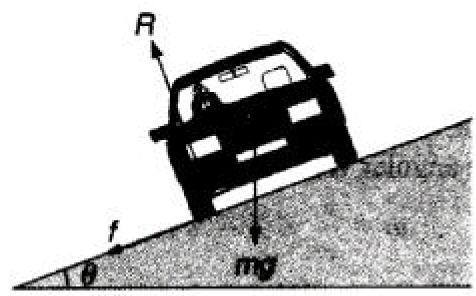
$$F = \frac{mv^2}{r}$$

$$= 800 \times \frac{15^2}{100}$$

$$= 1\ 800 \text{ N}$$

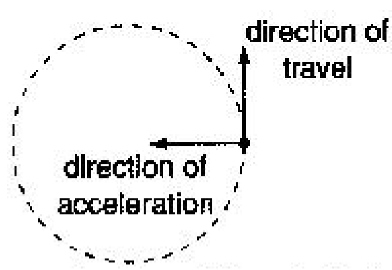
(ii) The centripetal force is provided by the static frictional force between the wheels and the ground.

(d) If the road is banked, the horizontal component of the normal reaction R provides part of the required centripetal force. This allows the car to move around a curve with higher speed. Therefore, banked road design is safer.



8. (a) (i) The direction of its velocity is always changing. Therefore, there is an acceleration even though the speed is constant.

(ii) The direction of the acceleration is always pointing towards the center.



(b) (i) $0.8mg = \frac{mv^2}{r}$

$r = 78.13\text{m}$

(ii) The centers of gravity of lorries are generally higher. The same sideways friction will produce a higher torque that may topple the lorry.

9. (d) **(i)** Speed of the moon in its orbit,

$$v = r\omega = r\frac{2\pi}{T}$$

$$= (3.84 \times 10^8) \times \frac{2\pi}{2.36 \times 10^6}$$

$$= 1\ 022 \text{ m s}^{-1}$$

(ii) Acceleraton of the moon,

$$a = \frac{v^2}{r} = \frac{1\ 022^2}{3.84 \times 10^8}$$

$$= 2.72 \times 10^{-3} \text{ m s}^{-2}$$

(iii) Force on the moon,

$$F = ma$$

$$= (7.35 \times 10^{22})(2.72 \times 10^{-3})$$

$$= 2.00 \times 10^{20} \text{ N}$$

10. (a) Angular velocity is the rate of change of angular displacement.

(b) (i) v = rù

(ii) v can be varied by varying r.

(iii) The tension in the cord provide the required centripetal force for circular motion.

(iv) $a = \dfrac{v^2}{r}$

$T = ma = m\dfrac{v^2}{r}$

$\therefore T = mv$ù

(c) (i) (1) $a = \dfrac{v^2}{r} = 20.6\text{ms}^{-2}$

(2) R + mg = ma

R = 634.2N

(ii) (1) change in PE = m g (2r) = 8400J (decreases)

(2) by conservation of energy,

$$8400 + \frac{1}{2}(60)(12)^2 = \frac{1}{2}(60)v^2$$

$v = 20.59\ \text{ms}^{-1}$

(iii) The entry speed must be sufficient for the cart and the passenger to reach the top of the loop at the speed where the centripetal force required is greater or equal to their weight. Otherwise, the cart and the passenger may fall off the track.

11. (a) (i) F_A is the friction exerted on the man by the wall and F_B is the normal reaction exerted on the man by the wall.

(ii) For the rotor to keep the man pinned against the wall,

$$F_A = mg = (70)(10) = 700\ \text{N}$$
$$F_A = 0.4\,F_B$$
$$F_B = \frac{700}{0.4} = 1750\ \text{N}$$

The normal reaction is the centripetal force keeping the man in a circular path.

$\therefore$ By $F_B = mr\omega^2$, $\omega^2 = \dfrac{1750}{70(2.5)} = 10$

$\omega = \sqrt{10} = 3.162\ \text{rad s}^{-1}$

$\therefore$ The minimum angular speed is 3.162 rad s^{-1}

(iii) Let the mass of the man be m.

Since $F_A = mg$, $F_B = \dfrac{mg}{0.4}$

$\because$ F_B = centripetal force

$$\frac{mg}{0.4} = mr\omega^2$$

$\omega = \sqrt{\dfrac{g}{0.4r}}$ which is independent of m

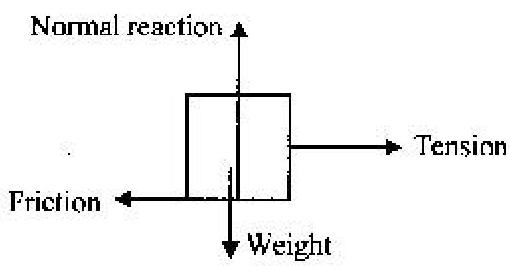
Therefore the result is unchanged.

(b) Since the space station is far from any planetary objects, there is negligible gravitational force acting on the astronauts. If the space station rotates about an axis through its center and normal with the plane containing the station with a constant angular speed, the normal reaction acting on the astronauts provides the centripetal force for keeping the astronauts in circular motion and this is experienced as 'gravity' felt by the astronauts. The same 'gravity' is felt by all people because it is independent of the mass of the astronauts.

$$a = r\omega^2$$
$$10 = (1 \times 10^3)\,\omega^2$$
$$\omega = 0.1 \text{ rad s}^{-1}$$

Therefore, rotating the space station with an angular speed of 0.1 rad s^{-1} can produce an artificial gravity of 10 N kg^{-1}.

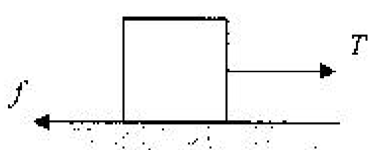
12. (a)



If the block is stationary, the $2m$ mass must be in equilibrium and hence $T = 2mg$. Consider all the horizontal forces, and a force F_{min} acts on the block to the left,

$$F_{min} = T - f_{min}$$
$$= 2mg - 0.6mg$$
$$= 1.4mg$$

(b) (i)

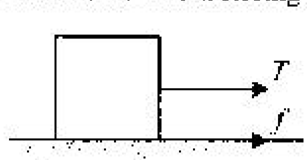


When ω is too small, the tension will not only provide centripetal force for circular motion, but also the force to make the block slide towards the center of rotation. In this case, friction acts in the opposite direction to the tension.

By $T - f = m\omega^2 r$, f will attain its maximum when ω is the smallest (but still performing circular motion). Moreover, $T = 2mg$ as explained in (a).

$$2mg - 0.6mg = m\omega_{min}^2 r$$
$$\omega_{min} = \sqrt{140} \text{ rad s}^{-1}$$

When ω is too large, both tension and friction will point towards the centre to prevent the block from sliding outward.



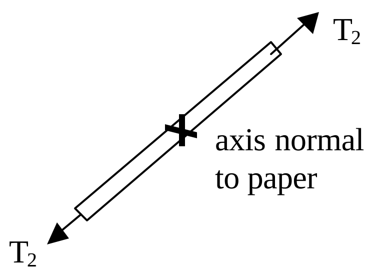
By $T + f = m\omega^2 r$, f will attain its maximum when ω is the largest (but still performing circular motion).

$$2mg + 0.6mg = m\omega_{max}^2 r$$
$$\omega_{max} = \sqrt{260} \text{ rad s}^{-1}$$

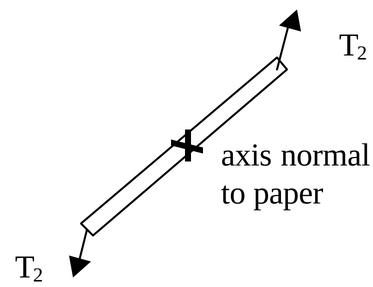
(ii) In between, T is a constant. (Note: Frictional force, initially pointing outwards, decreases gradually to zero and then its direction is reversed so as to accommodate the increase in ω).

(c) outside syllabus.

13. (a) As the rod does not rotate about an axis normal to paper, moment about its center must be zero.



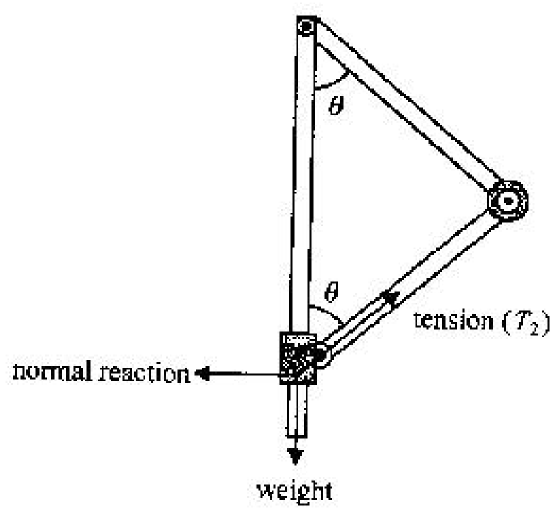
moment = 0



moment ≠ 0

Therefore, T_2 acts in a direction parallel to the lower rod.

Forces acting on the collar :



(b) (i) Consider the vertical forces acting on the collar:

$$T_2 \cos\theta = m'g \quad \text{——— (1)}$$

Consider the vertical forces acting on the counterweight:

$$T_1 \cos\theta = T_2 \cos\theta + mg \quad \text{— (2)}$$

Since the counterweight is performing (horizontal) uniform circular motion,

$$T_1 \sin\theta + T_2 \sin\theta = m\omega^2 L \sin\theta$$
$$T_1 + T_2 = m\omega^2 L \quad \text{——— (3)}$$

(ii) Substitute ② into ③,

$$\frac{T_2\cos\theta + mg}{\cos\theta} + T_2 = m\omega^2 L$$

$$\frac{mg}{\cos\theta} + 2T_2 = m\omega^2 L$$

Substitute ① into the above equation,

$$\frac{mg}{\cos\theta} + 2\cdot\frac{m'g}{\cos\theta} = m\omega^2 L$$

$$\omega^2\cos\theta = \frac{(m+2m')g}{mL}$$

$$= \left(1+\frac{2m'}{m}\right)\frac{g}{L}$$

(iii) If the mass m of the counterweight is increased while keeping the angular speed ω unchanged, $\cos\theta$ decreases. Therefore, θ increases and the collar moves up.

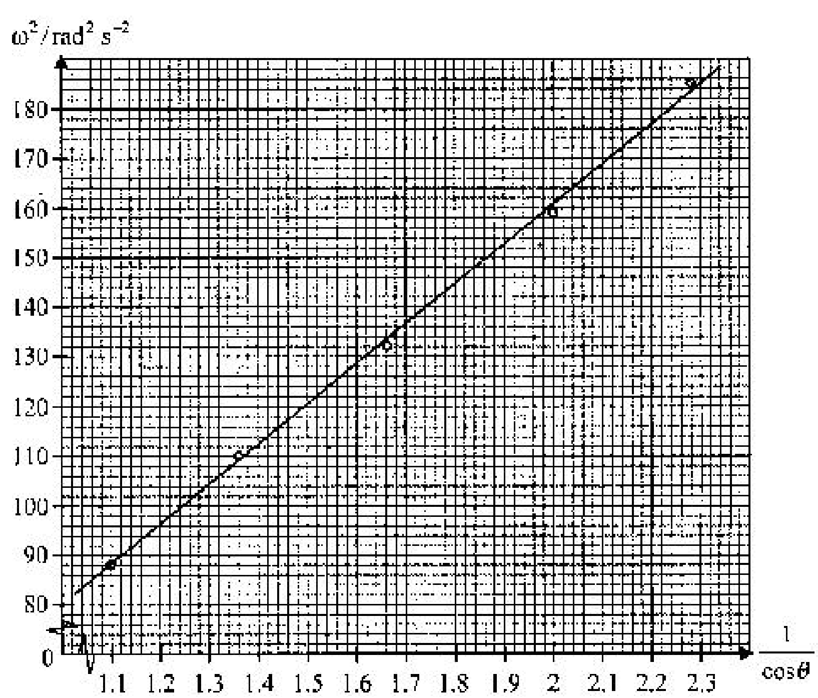
(c) From the previous calculation,

$$ù^2\,\text{cosè} = \left(1+\frac{2m'}{m}\right)\frac{g}{L}$$

$$ù^2 = \left[\left(1+\frac{2m'}{m}\right)\frac{g}{L}\right]\frac{1}{\text{cosè}}$$

The slope of the ω^2 against $\frac{1}{\cos\theta}$ is equal to $\left(1+\frac{2m'}{m}\right)\frac{g}{L}$.

ù / rad s^{-1}	13.6	12.6	11.5	10.5	9.4
θ	64 °	60 °	53 °	43 °	26°
ù^2 / rad^2 s^{-2}	184.96	158.76	132.25	110.25	88.36
$\frac{1}{\text{cosè}}$	2.28	2	1.66	1.35	1.11



Hence, slope of the graph = $\frac{130-88}{1.62-1.1} = \frac{1050}{13} = \left(1+\frac{2m'}{m}\right)\frac{g}{L}$

$$\frac{1050}{13} = \left(1+\frac{2m'}{m}\right)\frac{10}{0.5}$$

$$1+\frac{2m'}{m} = \frac{105}{26}$$

$$\frac{m'}{m} = \frac{3}{2}$$