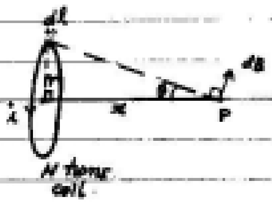
Field at an axial point.



$$dB = N \frac{\mu_0 i}{4\pi} \frac{dl}{r^2 \csc^2\theta}$$

Component of dB along axis of the coil

$$dB_{axis} = dB \sin\theta$$

$$= \frac{N \mu_0 i \sin^3\theta}{4\pi r^2} dl$$

$$\therefore \quad B = \int dB_{axis}$$

$$= \int \frac{N \mu_0 i \sin^3\theta}{4\pi r^2} dl$$

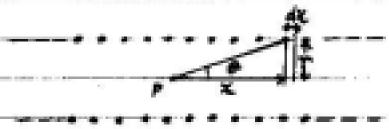
$$= \frac{N \mu_0 i \sin^3\theta}{4\pi r^2} [2\pi r]$$

$$= \frac{N \mu_0 i \sin^3\theta}{2r}$$

Field inside a very long solenoid.

number of turns in the element 'dx'

$$dN = n\, dx$$



n = turn density = number of turns per metre

$$dB = \frac{\mu_0 i \sin^3\theta}{2r} dN$$

$$= \frac{\mu_0 i \sin^3\theta\, n}{2r} dx$$

$$x = r \cot\theta$$

$$dx = -r \csc^2\theta \; d\theta$$

$$\therefore \quad dB = \frac{\mu_0 i \sin^3\theta\, n}{2r} (-r \csc^2\theta \; d\theta)$$

$$= \frac{-\mu_0 i n}{2} \sin\theta \; d\theta$$

$$\therefore \quad B = \int dB$$

$$= \int_\pi^0 \frac{-\mu_0 i n}{2} \sin\theta \; d\theta$$

(note: $x \to -\infty$, $\theta \to \pi$ and $x \to \infty$, $\theta \to 0$)

$$= \frac{\mu_0 i n}{2} [\cos\theta]_\pi^0$$

$$\therefore \quad \boxed{B = \mu_0 n i}$$